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Topology and broken Hermiticity

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Topology and symmetry have emerged as compelling guiding principles to predict and harness the propagation of waves in natural and artificial materials. Be it for quantum particles (such as electrons) or classical waves (such as light, sound or mechanical motion), these concepts have so far been mostly developed in idealized scenarios, in which the wave amplitude is neither attenuated nor amplified, and time evolution is unitary. In recent years, however, there has been a considerable push to explore the consequences of topology and symmetries in non-conservative, non-equilibrium or non-Hermitian systems. A plethora of driven artificial materials has been reported, blurring the lines between a wide variety of fields in physics and engineering, including condensed matter, photonics, phononics, optomechanics, as well as electromagnetic and mechanical metamaterials. Here we discuss the latest advances, emerging opportunities and open challenges for combining these exciting research endeavours into the new pluridisciplinary field of non-Hermitian topological systems.

The study of non-conservative topological systems has recently seen an explosion of activity, due to the intriguing new physics occurring at the boundary between two active fields of research: non-Hermitian systems and topological phenomena. Non-Hermitian systems, on the one hand, can be defined generally as systems governed by a dynamic equation of the Schrödinger form $i\partial_t|\psi> = H|\psi>$, where t is the time variable, $i=\sqrt{-1}$ and $|\psi>$ is a vector describing a wave function (in quantum mechanics) or a wave field (in mechanics or optics), and where the time-evolution operator H is non-Hermitian. One of the consequences of the non-Hermiticity of H is that the amplitude $<\psi|\psi>$ is no longer conserved. In quantum physics, this implies a lack of probability conservation, or a loss of quantum information to the environment. For classical-wave phenomena, be it of electromagnetic or mechanical nature, the state $|\psi>$ is generally defined such that $<\psi|\psi>$ is proportional to the energy[1]. Thus, non-Hermiticity is related to a lack of conservation of wave energy. For example, non-Hermitian wave components are readily found in optics with structures containing optical gain or absorption losses and in mechanics with structures containing electromechanical transducers or dissipation. Over the past decade, the study of the interplay of amplification, damping and the coupling between different optical components has given rise to the large field of non-Hermitian photonics, which deals with the development of new devices that leverage gain and loss as a new degree of freedom. Fascinating modal and scattering properties of non-Hermitian systems have been uncovered, such as the ones of parity–time symmetric systems[2–4], and systems operated near or encircling exceptional points[5–7]. Indeed, due to the non-Hermitian nature of H, time-harmonic modal solutions are associated with a non-Hermitian eigenproblem $H|\psi> = \omega|\psi>$ whose eigenvalues are, in general, complex, and whose eigenmodes do not generally form a complete basis. This leads to markedly new physics, even in loss-only non-Hermitian systems; gain comes into play when one wants to compensate the damping of the energy in time, or to reach instabilities.

On the other hand, topological physics has attracted a lot of attention recently, from its origins in condensed-matter physics[8], to its adaptation in a wide range of artificial-wave systems including photonics[9,10], acoustics[11] and mechanics[12].

The topological properties of these systems are invariant under smooth deformations, such as geometrical or material parameter changes. This behaviour is characterized by a global integer quantity — called a topological invariant — that can change only through a discontinuous modification of the system. The associated physical properties therefore inherit some form of topological protection, as they are guaranteed to survive any continuous perturbation. In the Hermitian setting, topology has led the way to various protected features, including robust boundary waves in periodic, quasicrystalline[13], fractal[14], amorphous and structure-free systems[15–17], topologically protected scattering signatures[18,19], or other forms of sturdy modes such as bound states in continuum[20] or unidirectional radiation channels[21]. More recently, a large variety of exciting developments has emerged based on more complex systems with broken Hermiticity or time invariance, including topological time-driven Floquet quantum or classical systems[22–25], systems with hidden or synthetic dimensions[26,27], topological non-Hermitian systems[28] and topological lasers[29–32].

In this Perspective, we propose a classification of wave systems that allows us to highlight the direction of the field of classical-wave topology, which is currently moving towards a merger with non-Hermitian systems. The classification helps to identify the various mechanisms that have been used to engineer non-trivial topologies in Hermitian and non-Hermitian systems, and highlights the new physical properties that have been reported. We identify different promising research directions for the new field of non-Hermitian topological systems.

Towards non-Hermitian topology

In this section, we propose a classification of topological systems based on the concepts of preserved or broken Hermicity and time invariance.

Time invariance and Hermiticity. Although topological waves have been reported in very different physical systems and experimental platforms, the current research landscape in the field can be described using a classification focused on two basic properties: time invariance and Hermiticity (Fig. 1). In time-invariant systems, the physical properties that dictate wave propagation (for example, index of refraction, geometry, composition and so on) do not vary

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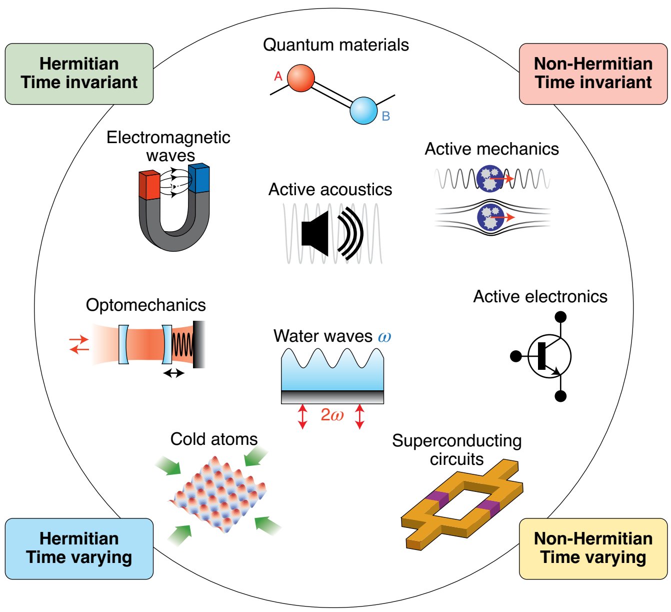

Fig. 1 | Classification of topological systems and examples of strategies to engineer symmetries and/or break Hermiticity. The classification comprises four quadrants, (1) Hermitian and time-invariant systems; (2) non-Hermitian and time-invariant systems; (3) Hermitian and time-varying systems; and (4) non-Hermitian and time-varying systems. The examples are a non-exhaustive list of possible wave-based platforms: quantum materials, depicted as a tight-binding model; electromagnetic waves, represented by a magnetic field; active mechanics and acoustics, in which momentum and/or energy can be injected by motorized particles or loudspeakers; optomechanical devices, depicted by a laser cavity coupled to mechanical motion; water waves close to a parametric instability, in which waves at a frequencies ω are being pumped by a periodic drive at twice the frequency 2ω; active electronics, depicted by a transistor; cold gases under laser pumping; and superconducting circuits.

in time, leading to a time-independent Hamiltonian. Hermiticity on the other hand, forbids non-conservative exchange of energy between a wave and its medium of propagation. In other words, Hermitian systems are always conservative from the viewpoint of the wave, which does not lose or gain energy while propagating, whereas in non-Hermitian systems, the waves can be amplified and/or damped. In non-Hermitian systems, reversing time typically implies turning gain into loss and vice versa, and so time-reversal symmetry is broken. Interestingly, time invariance and Hermiticity can be changed independently in a given system. All four combinations of time-invariant/time-varying and Hermitian/non-Hermitian features can be realized in practice. This allows us to propose a classification, as represented by the four quadrants in Fig. 1.

Examples of time-invariant wave systems include any situation in which the medium can be considered as static from the viewpoint of the wave. For instance, standard photonic crystals or metamaterials, based on material mixtures or material architectures, are time invariant. These systems can be Hermitian or non-Hermitian. The latter correspond to the presence of static losses or gain, like in the field of non-Hermitian photonics[33]. In an optical gain medium, some internal dynamics is used to create the required population inversion. However, from the point of view of the wave, this process can be considered as time invariant: population inversion is continuously maintained into a steady state by the external pump. In mechanical or acoustic systems, time-invariant gain can be similarly applied using sensors and actuators to implement active-control systems that react faster than the wave[34,35].

Time-varying systems can also be either Hermitian or non-Hermitian, depending on the timescales involved. Indeed, adiabatic variation of a system, defined as time variations much slower than the period of the wave, can lead to geometric phase effects[36], but not to energy exchanges, due to the size of frequency mismatch. Similarly, material variations, when much faster than the wave oscillation, generally lead to effects that average to Hermitian interactions[23].

In contrast, when material variations occur on a timescale that is of the same order of magnitude as the wave's period, parametric amplification is possible, and strong non-Hermiticity can be induced. This leads to the category of time-varying non-Hermitian systems. A simple example of parametric amplification is a person standing on a swing, and flexing their legs at twice the swing's natural resonance frequency, effectively modulating its length in time. Such a parametric system is non-Hermitian, as the amplitude of the motion of the swing increases exponentially with time. The example of the swing can be mathematically expressed by noting that, in a parametrically driven system, the Hamiltonian is periodic in time. The use of the Floquet theorem allows the definition of the stroboscopic (Floquet) Hamiltonian, which necessarily has complex eigenvalues with a non-zero positive imaginary part (otherwise the system would not parametrically amplify over time). As a mathematical consequence, the stroboscopic Hamiltonian is therefore necessarily non-Hermitian. This is how one can link parametric systems to non-Hermiticity (and define Floquet exceptional points and so on). More complex cases are found in various systems, for example, for water[37] and electromagnetic[38,39] waves.

The classification based on time invariance and Hermiticity is not exhaustive. In particular, nonlinearities or interactions could perhaps provide a third axis in the diagram shown in Fig. 1. Although interactions will undoubtedly enrich the range of physical phases and properties that may be accessed, we focus on non-interacting or linear systems, and show that even there, much remains to be explored.

Mechanisms used to engineer topology. It is interesting to use the above classification as a reading grid to understand how the concept of topology has developed in these different kinds of system, and which areas it is likely to develop towards in the near future. As summarized in Fig. 1, different mechanisms have been employed to leverage or break symmetries in artificial materials and engineer non-trivial topological phases.

Time-invariant and Hermitian systems. The most traditional designs are found in the category of time-invariant Hermitian systems. In particular, they include conventional Chern insulators, which require breaking time-reversal symmetry by externally biasing the system with a time-odd quantity, such as an external magnetic field polarizing a magneto-photonic crystal[40,41]. In acoustics or mechanics, time-reversal symmetry can be violated by imparting a steady flow or motion[42–45]. In this category, we also find the different class of Z_2 insulators, which are based on preserved time-reversal symmetry[46]. Despite the lack of intrinsic spin degrees of freedom in classical-wave physics, decoupled degenerate time-reversed modes (called pseudospins) can be engineered based on a geometrical symmetry or duality property[10,12,47–49], or accidental band degeneracy[50]. Once the pseudospin states are established, other compatible symmetries may be exploited or broken, such as mirror, inversion or rotational symmetries, to open bandgaps and drive topological phase transitions. Note that the time-reversal operator has a different definition for different systems, and is defined differently for electrons and photons[9].

From these traditional approaches, leveraging standard mechanisms, other topological concepts have emerged, using active mechanisms that couple local degrees of freedom of the artificial material

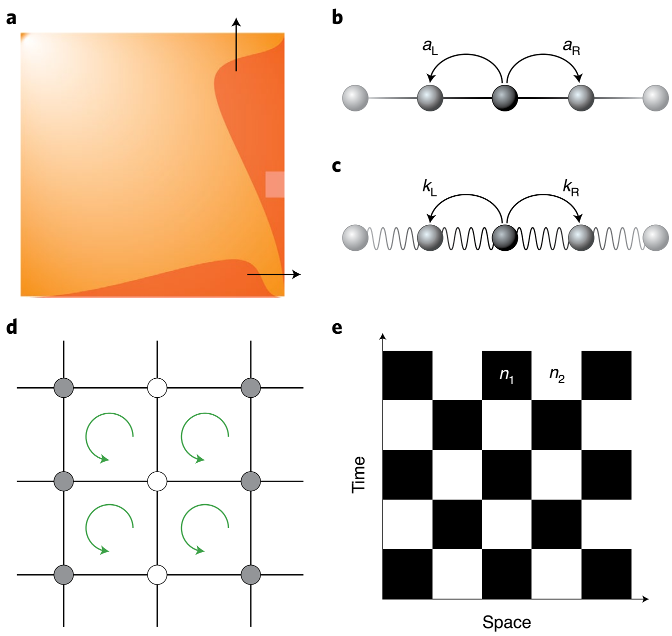

Fig. 2 | Emergence of topology and broken symmetry in non-Hermitian materials. a, Topological lasing/amplification in topological modes[32,68]. The material is depicted in light orange and the self-amplifying waves propagating in the direction of the arrows are shown in dark orange. **b**, Hatano–Nelson model of an atomic chain, where the hopping probabilities a_L and a_R of jumping to the left and to the right differ[28]. **c**, Mechanical counterpart, where the right-to-left and left-to-right stiffnesses k_L and k_R differ[35]. **d**, Hofstadter model, implemented in cold atoms via external time-modulated excitations[23]. The grey and white dots represent two different potential wells. The green arrows denote the intrinsic chirality in each plaquette of 2 × 2 lattice sites. **e**, Example of a time-modulated non-Hermitian material, where the materials' indices of refraction, n_1 and n_2, alternate in both space and time, leading to self-amplifiying waves[67].

to the outside. Just as transistors have revolutionized electronics, active meta-atoms are emerging as an exquisite platform to manipulate waves in an unprecedented fashion via suitably engineered symmetries. Such systems based on active building blocks make up the three other classes of system in Fig. 1.

Time-invariant and non-Hermitian systems. The first of the three classes of active system keeps the property of time invariance but breaks Hermiticity. In this case, non-Hermiticity comes as an extra degree of freedom to engineer a new functionality or induce a non-trivial topology. An example of the former is the topological laser (Fig. 2a), in which a topological system is turned into a laser by adding gain media, nonlinear saturation mechanisms and a radiation loss channel[32]. In other cases, the non-Hermiticity is itself essential to obtaining non-trivial topology, for instance, in the one-dimensional Hatano–Nelson chain[28], which is a tight-binding toy model involving non-reciprocal, non-Hermitian hopping terms (Fig. 2b,c). Recent theoretical advances and experimental implementations of various flavours of this model have highlighted new non-Hermitian phenomena, which can be related to topology, but are not always[35,51–57]. A prime example of a purely non-Hermitian phenomenon that is not related to topology is the non-Hermitian skin effect[51,52,58], wherein the wave spectrum is extremely sensitive to boundary conditions and where in particular, all modes localize on one edge for open boundaries. This effect has been experimentally reported recently in mechanical systems[35], in active electrical circuits[59] and in quantum systems[53]. Finally, purely non-Hermitian topological invariants and novel forms of non-Hermitian topological bulk–boundary correspondences have recently been proposed[52,54,55,60] and verified experimentally[55], but the topic remains a matter of intense debate[54,56,57]: the normally straightforward correspondence between a spectral gap closing in periodic systems and a simultaneous change in the number of edge states in open systems known from Hermitian topological insulators breaks down in non-Hermitian settings[61,62]. Different types of bulk–edge correspondence have been proposed as generalizations of this concept, but their theoretical formulations and experimental exploration are still in their initial stage[52,54–57,59].

Time-varying and Hermitian systems. Mechanisms used to engineer topology in systems whose properties depend on time are shown in Fig. 1. The vast majority belong to the so-called Floquet systems[63,64], in which the time dependence is periodic with a modulation frequency ω_m that is generally different to that of the propagating wave ω. In the adiabatic regime $\omega_m << \omega$, the frequency mismatch between the modulation and the wave prevents energy exchange between them, and the system can be considered Hermitian. Such adiabatic time modulation can lead to geometrical phase effects that may be exploited to break time-reversal symmetry and induce various topological effects, for example, in photonics or phononics[24,26,27,65]. If the modulation is fast compared with the mode propagation, $\omega_m >> \omega$, it is often possible to average the effect of modulation, leading again to an effectively time-invariant Hermitian system, which may still have topological properties. For instance, the Hofstadter Hamiltonian has been emulated with fast time-dependent systems, such as cold atoms in dynamic optical lattices[23] (Fig. 2d). Their mechanical equivalents have been theoretically studied[25] but not yet experimentally demonstrated.

Time-varying and non-Hermitian systems. Finally, to the best of our knowledge, the implementation of topological time-varying non-Hermitian systems belonging to the fourth quadrant of Fig. 1 has only just begun[66]. Systematic steps into this new arena are currently being taken: non-adiabatic temporal changes can be leveraged to control waves[37], time-modulated systems could exhibit novel forms of self-amplifying wave propagation[39,67,68] (Fig. 2e) and topology could be obtained by modulating non-Hermitian parameters in time pumping[68,69]. However, in time-modulated non-Hermitian systems, there can be no separation of scales between the temporal profile of the impinging wave and that of the time modulation. This challenges the formulation of perturbative approaches, which typically rely on separation of timescales and thus complicates the identification of effective non-Hermitian Hamiltonians with well-defined topological properties. For this reason, conceptual progress is needed to unify the various platforms and predict any new physics that may emerge. In addition, the geometrical phase effects obtained by encircling exceptional points via adiabatic spatial modulations[6,7] could also be observed using temporal modulations. Based on the many new ways of manipulating geometrical phases when Hermitian systems are allowed to be driven in time, and on the emergence of fundamentally new types of topology when time-invariant systems are allowed not to be Hermitian, one may expect many more surprises when both constraints are lifted simultaneously.

Open challenges and new directions

The classification we propose allows us to identify uncharted territory and to classify excursions into it. There has recently been a lot of activity involving time-invariant-Hermitian and time-varying-Hermitian systems. In these areas, open challenges remain, for example, to obtain a complete picture of the interplay between lattice symmetries and topological phases[70–73], exploit higher-order topological phases[74] or to utilize different types of topology in applications, for example, by scaling down towards on-a-chip platforms[75] or leveraging amorphous systems[16]. While topological classical-wave systems allow for robust point-to-point energy guiding with an engineered immunity to disorder or geometrical imperfections, they are still inherently sensitive to absorption losses. Therefore, extending them to the non-Hermitian regime may allow the balancing of absorption with gain, creating some form of sturdy constant-amplitude topological waves[34], or making topological edge states robust to both Hermitian and non-Hermitian defects. Above the lasing threshold, the interplay between topological non-Hermitian modes and nonlinear saturation phenomena may bring about unexpected physical effects, for instance, self-adaptation of the energy transport[76]. Very few studies have started to explore how topological concepts translate to the non-Hermitian realm, focusing mainly on physics-driven explorations[54], or experimental validations of canonical one-dimensional scenarios[53,55,59]. The extension of non-Hermitian topological concepts to two dimensions is so far largely restricted to situations in which a non-trivial topology is already present in the Hermitian limit[32,60]. Yet, these pioneering works suggest that fundamental questions are still open, for example, pertaining to the generality and robustness of the non-Hermitian bulk–edge correspondence and its generalization and classification in higher dimensions. In addition, the class of topological systems that are time-varying and non-Hermitian has been left virtually unexplored, whereas a few experimental platforms may soon become sufficiently mature to investigate these directions. All these potential directions suggest that big opportunities to discover new physics and applications lie at the boundary between the fields of topological and non-Hermitian systems.

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Competing interests
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